

The Effect of Time Variation in the Higgs Vacuum Expectation Value on the Cosmic Microwave Background

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A time variation in the Higgs vacuum expectation value alters the electron mass and thereby changes the ionization history of the universe. This change produces a measurable imprint on the pattern of cosmic microwave background (CMB) fluctuations. The nuclear masses and nuclear binding energies, as well as the Fermi coupling constant, are also altered, with negligible impact on the CMB. We calculate the changes in the spectrum of the CMB fluctuations as a function of the change in the electron mass m_e . We find that future CMB experiments could be sensitive to $|\Delta m_e/m_e| \sim |\Delta G_F/G_F| \sim 10^{-2} - 10^{-3}$. However, we also show that a change in m_e is nearly, but not exactly, degenerate with a change in the fine-structure constant α . If both m_e and α are time-varying, the corresponding CMB limits are much weaker, particularly for $l < 1000$.

I. INTRODUCTION

The possibility that the fundamental constants of nature are not, in fact, constant, but might vary with time has long been an object of speculation by physicists [1]. The fundamental constants which have received the greatest attention in this regard are the coupling constants which determine the interaction strengths of the fundamental forces: the gravitational constant G , the fine-structure constant α , and the coupling constants for the weak and strong interactions. It has recently been noted that measurements of the cosmic microwave background (CMB) fluctuations in the near future will sharply constrain the variation of α at redshifts ~ 1000 [2,3]; here we extend this analysis to the Fermi coupling constant, through its dependence on the Higgs vacuum expectation value.

As emphasized by Dixit and Sher [4] (see also reference [5]) the Fermi constant is not a fundamental coupling constant; it is actually independent of the gauge coupling constant and depends directly on the Higgs vacuum expectation value $\langle\phi\rangle$: specifically, $G_F \propto \langle\phi\rangle^{-2}$. Hence, it is most meaningful to discuss constraints on the time variation of $\langle\phi\rangle$, rather than G_F . Furthermore, the possibility of a time-variation in the vacuum expectation value of a field seems more plausible than the time variation of a fundamental coupling constant. (For more detailed arguments in favor of considering (spatial) variations in $\langle\phi\rangle$, see reference [6]).

Constraints on the time variation of G_F or $\langle\phi\rangle$ have been considered previously in references [4,5,7]. As noted in reference [4], changing $\langle\phi\rangle$ has four main physical effects with astrophysical consequences: G_F changes, the electron mass m_e changes, and the nuclear masses and binding energies change. All four of these alter Big Bang nucleosynthesis, and requiring consistency with the observed element abundances gives limits of [5] $\Delta G_F/G_F < 20\%$ at a redshift on the order of 10^{10} . In contrast, only one effect is relevant for the CMB spectrum: the change in m_e . The weak interactions have no relevance at the epoch of recombination, while the effect of changing the nuclear masses and binding energies is negligible compared to the effect of altering m_e . Hence, for the purposes of the CMB, we can treat a change in the Higgs vacuum expectation value as equivalent to a change in m_e alone, where $m_e \propto \langle\phi\rangle$.

In the next section, we describe the changes in recombination produced by a change in m_e and show how the CMB fluctuation spectrum is altered. We also examine the degeneracy between altering m_e and changing the fine structure constant α . In Sec. III, we translate our results into limits on a time-variation in m_e and, therefore, on the variation of $\langle\phi\rangle$ and G_F . We find that the MAP and PLANCK experiments might be sensitive to variations as small as $|\Delta m_e/m_e| \sim 10^{-2} - 10^{-3}$, although the limits are much weaker if α is allowed to vary as well.

II. CHANGES IN THE RECOMBINATION SCENARIO AND THE CMB

As in references [2,3], we will assume that the variation in m_e is sufficiently small during the process of recombination that we need only consider the difference between m_e at recombination and m_e today; i.e., we treat m_e as constant during recombination. The electron mass m_e changes the CMB fluctuations because it enters into the expression for the differential optical depth $\dot{\tau}$ of photons due to Thomson scattering:

$$\dot{\tau} = x_e n_p c \sigma_T, \quad (1)$$

where σ_T is the Thomson scattering cross-section, n_p is the number density of electrons (both free and bound) and x_e is the ionization fraction. The Thomson cross section depends on m_e through the relation

$$\sigma_T = 8\pi\alpha^2\hbar^2/3m_e^2c^2. \quad (2)$$

The dependence of x_e on m_e is more complicated; it depends on both the change in the binding energy of hydrogen:

$$B = \alpha^2 m_e c^2 / 2, \quad (3)$$

which is the dominant effect, and also on the change in the recombination rates with m_e . Note that m_e and α enter into the expressions for B and σ_T in different ways, so that the effect of changing m_e cannot be parametrized in a simple way in terms of the effect of changing α (calculated in references [2,3]). However, since the change in B dominates all other effects, we expect significant degeneracy between the effect of changing m_e and the effect of changing α . Since a change in m_e affects the same physical quantities as a change in α , our discussion will parallel that in reference [3].

The ionization fraction x_e is determined by the ionization equation for hydrogen [8,9]:

$$-\frac{dx_e}{dt} = \mathcal{C} \left[\mathcal{R} n_p x_e^2 - \beta(1 - x_e) \exp\left(-\frac{B_1 - B_2}{kT}\right) \right], \quad (4)$$

where \mathcal{R} is the recombination coefficient, β is the ionization coefficient, B_n is the binding energy of the n^{th} hydrogen atomic level and n_p is the sum of free protons and hydrogen atoms. The Peebles correction factor \mathcal{C} accounts for the effect of non-thermal Lyman- α resonance photons and is given by:

$$\mathcal{C} = \frac{1 + A}{1 + A + C} = \frac{1 + K\Lambda(1 - x_e)}{1 + K(\Lambda + \beta)(1 - x_e)}, \quad (5)$$

where $K = H^{-1} n_p c^3 / 8\pi\nu_{12}^3$ (ν_{12} is the Lyman- α transition frequency), and Λ is the rate of decay of the 2s excited state to the ground state via 2 photons and scales as m_e [10]. Since ν_{12} scales as m_e , we have $K \propto m_e^{-3}$. The ionization and recombination coefficients are related by the principle of detailed balance:

$$\beta = \mathcal{R} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \exp\left(-\frac{B_2}{kT}\right), \quad (6)$$

and the recombination coefficient can be expressed as

$$\mathcal{R} = \sum_{n,\ell}^* \frac{(2\ell+1)8\pi}{c^2} \left(\frac{kT}{2\pi m_e} \right)^{3/2} \exp\left(\frac{B_n}{kT}\right) \int_{B_n/kT}^{\infty} \frac{\sigma_{n\ell} y^2 dy}{\exp(y) - 1}, \quad (7)$$

where $\sigma_{n\ell}$ is the ionization cross-section for the (n, ℓ) excited level of hydrogen [11]. In the above, the asterisk on the summation indicates that the sum from $n = 2$ to ∞ needs to be regulated. The m_e dependence of the ionization cross-section is rather complicated, but can be written as $\sigma_{n\ell} \sim m_e^{-2} f(h\nu/B_1)$, from which one can derive the following equation:

$$\frac{\partial \mathcal{R}(T)}{\partial m_e} = -\frac{1}{m_e} \left(2\mathcal{R}(T) + T \frac{\partial \mathcal{R}(T)}{\partial T} \right). \quad (8)$$

This equation allows us to relate the m_e dependence of the recombination coefficient to its temperature parametrization $\mathcal{R}(T)$, which can be approximated by a power law of the form $\mathcal{R}(T) \sim T^{-\xi}$. Then a solution of equation (8) has the m_e dependence $\mathcal{R} \propto m_e^{\xi-2}$. As in reference [3] we will take $\xi = 0.7$, corresponding to power law $\mathcal{R}(T) \sim T^{-0.7}$. We are interested in small changes in m_e , so that $m'_e = m_e(1 + \Delta_m)$ with $\Delta_m \ll 1$. Now equation (4) including a change in m_e can be written as:

$$-\frac{dx_e}{dt} = \mathcal{C}' \left[\mathcal{R}' n_p x_e^2 - \beta'(1 - x_e) \exp\left(-\frac{B'_1 - B'_2}{kT}\right) \right], \quad (9)$$

with $\mathcal{R}' = \mathcal{R}(1 + \Delta_m)^{\xi-2}$, the changed binding energies $B'_n = B_n(1 + \Delta_m)$,

$$\beta' = \beta(1 + \Delta_m)^{\xi-1/2} \exp\left(-\frac{B_2 \Delta_m}{kT}\right), \quad (10)$$

and the changes in the Peebles factor (equation 5). We then integrated equations (9) and (1) using CMBFAST [12] to obtain the CMB fluctuation spectra for different values of m_e .

Fig. 1 shows the results for a change in m_e of $\pm 5\%$ for a standard cold dark matter model (SCDM) with $h = 0.65$ and $\Omega_b h^2 = 0.02$. There are two main effects, similar to what is seen for a change in the fine-structure constant [3]. First, an increase in m_e shifts the curves to the right (i.e., larger l values) due to the increase in the hydrogen binding energy, which results in earlier recombination, corresponding to a smaller sound horizon at the surface of last scattering. Second, the amplitude of the curves increases with increasing m_e . This second change is due to two different physical effects: an increase in the early ISW effect due to earlier recombination (which dominates at small l) and a change in the diffusion damping (which dominates at large l) [3].

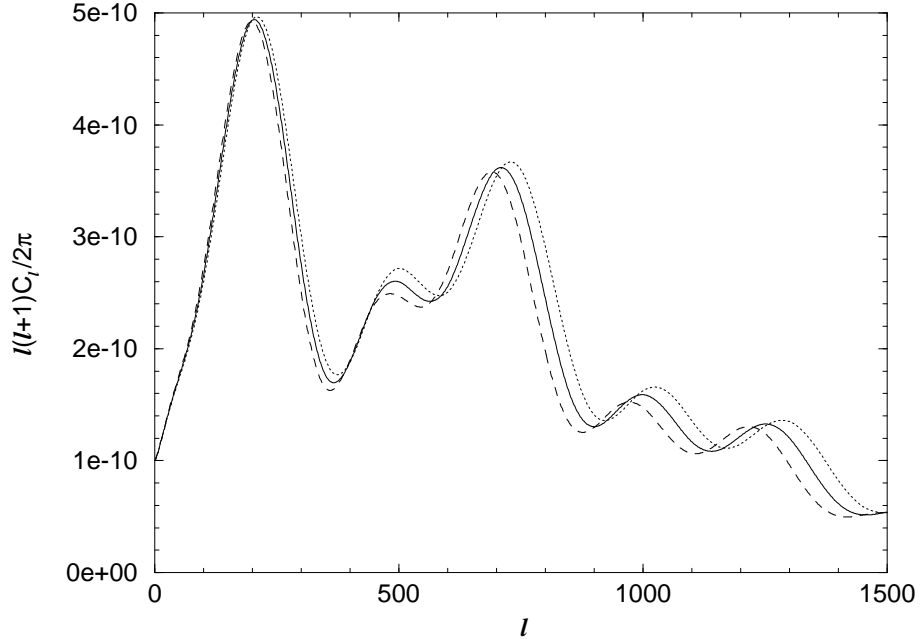


FIG. 1. Spectrum of CMB fluctuations for a standard cold dark matter scenario (SCDM, $\Omega_b h^2 = 0.02$, $h = 0.65$) for no change in the electron mass (solid curve) and for a change in m_e of $\pm 5\%$: $\Delta_m = +5\%$ (dotted curve) and $\Delta_m = -5\%$ (dashed curve).

Since a change in α affects the same physical quantities as a change in m_e , it is not surprising that the effects on the CMB fluctuation spectrum are similar. However, they are not identical. This can best be illustrated by choosing changes in m_e and α which leave the binding energy B unchanged, i.e., $(1 + \Delta_\alpha)^2(1 + \Delta_m) = 1$, since the change in B dominates the changes in the fluctuation spectrum. This is illustrated in Fig. 2, in which we have taken a 3% increase in α and a 5.74% decrease in m_e .

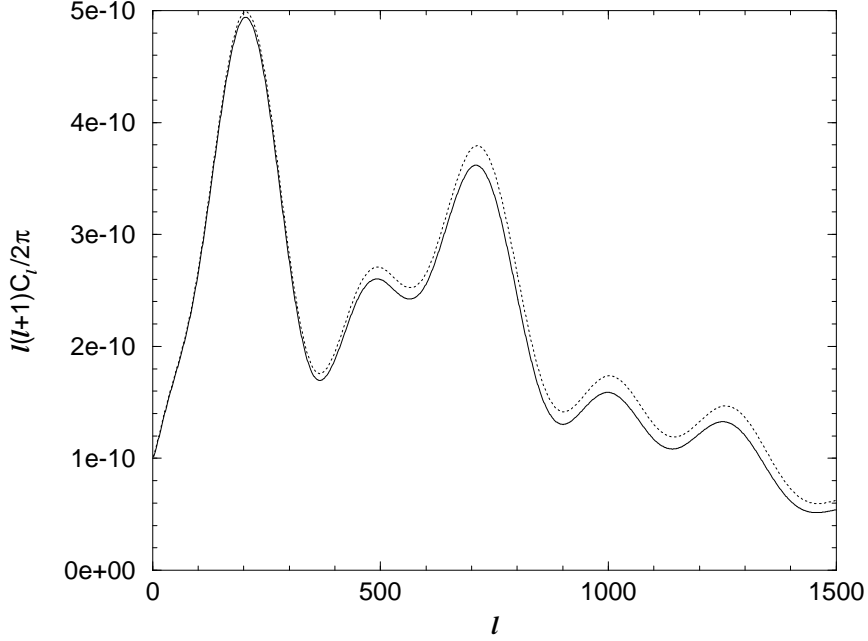


FIG. 2. Spectrum of CMB fluctuations for a standard Λ CDM model ($\Omega_b h^2 = 0.02$, $h = 0.65$) (solid curve) and for a change in both α and m_e , $\Delta_\alpha = +3\%$ and $\Delta_m = -5.74\%$, which leaves the hydrogen binding energy unchanged (dotted curve).

As expected, the changes in m_e and α nearly cancel in their effect on the CMB, and there is no shift in the location of the peaks. However, there is a residual increase in the amplitude which is largest at large l . Recall that the shift in the position of the peaks and the change in their amplitude at small l are dominated by the change in the binding energy, which is zero in this case. However, the change in the diffusion damping, which dominates the change in the amplitude at large l , scales differently with m_e and α , producing an increase in the peak amplitude at large l . If both α and m_e are assumed to be variable, any CMB constraints on this variation will be considerably weaker. There is some theoretical justification to consider such models [4,7].

III. LIMITS ON VARIATIONS IN THE ELECTRON MASS

We know from the analysis in references [2,3] and in the previous section that variations in α and/or m_e will change the CMB spectrum significantly. In order to impose limits on this variation from future CMB data, the Fisher information matrix is a very useful tool. For small variations in the parameters (θ_i) of a cosmological model the likelihood function (\mathcal{L}) can be expanded about its maximum as

$$\mathcal{L} \simeq \mathcal{L}_m \exp(-F_{ij} \delta\theta_i \delta\theta_j), \quad (11)$$

where F_{ij} is the Fisher information matrix, as defined in reference [13]

$$F_{ij} = \sum_{\ell=2}^{\ell_{\max}} \frac{1}{\Delta \mathcal{C}_\ell^2} \left(\frac{\partial \mathcal{C}_\ell}{\partial \theta_i} \right) \left(\frac{\partial \mathcal{C}_\ell}{\partial \theta_j} \right), \quad (12)$$

where $\Delta \mathcal{C}_\ell$ is the error in the measurement of \mathcal{C}_ℓ . In this approximation the inverse of the Fisher matrix F^{-1} is the covariance matrix, and in particular the variance of parameter θ_i is given by $\sigma_i^2 = (F^{-1})_{ii}$. In the case of the CMB the cosmological parameters (θ_i) that are taken to be determined from the measured fluctuation spectrum are the Hubble parameter h , the number density of baryons (parametrized as $\Omega_b h^2$), the cosmological constant (parametrized as $\Omega_\Lambda h^2$), the effective number of relativistic neutrino species N_ν , and the primordial helium abundance Y_p . Additionally, we allow the electron mass m_e to serve as an undetermined parameter, and consider also the effect of adding the fine-structure constant α to this set. We make the assumption that the experiments are limited only by the cosmic

variance up to a maximum ℓ , denoted by ℓ_{max} . Analysis of the Fisher information matrix will now enable us to calculate a rough upper bound for the limits on Δ_m which could be obtained from future CMB experiments.

We analyze two flat ($\Omega = 1$) cold dark matter models, a standard cold dark matter model (SCDM) and one with $\Omega_\Lambda = 0.7$ (Λ CDM). Both models have $h = 0.65$, $\Omega_b h^2 = 0.02$, $N_\nu = 3.04$ and $Y_p = 0.246$. For each model we calculate the variation in the electron mass, σ_m/m , as a function of ℓ_{max} for two different cases. In the first case we consider only m_e to vary, taking α as constant; in the second case we take both m_e and α to be variable. The results are shown in Fig. 3.

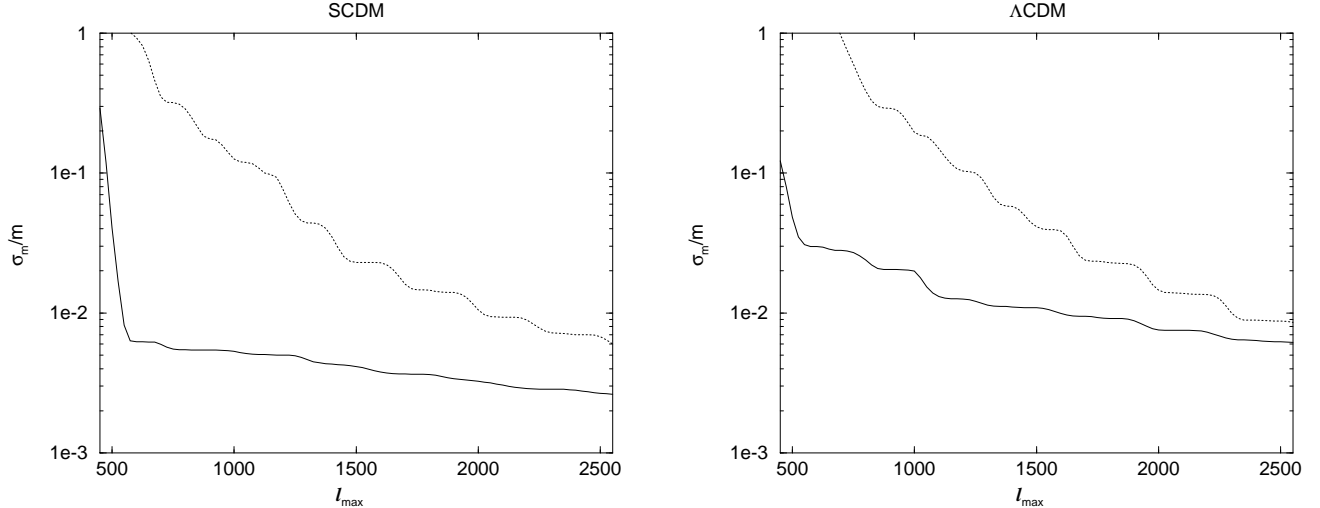


FIG. 3. The estimated accuracy with which variations in m_e could be constrained by a future CMB experiment, as a function of the maximum angular resolution given by ℓ_{max} for two cases: m_e variable, but α constant (solid curves), and both m_e and α variable (dotted curves).

If α is taken to be constant, the upper limits on $|\Delta m_e/m_e|$ are of order $10^{-2} - 10^{-3}$ for $\ell_{max} \sim 500 - 2500$ in both the SCDM and Λ CDM models. Since $m_e \propto \langle \phi \rangle$, and $G_F \propto \langle \phi \rangle^{-2}$, similar limits apply to the variation in $\langle \phi \rangle$ and G_F . This represents potentially a much tighter limit on the time variation in G_F than can be obtained from Big Bang nucleosynthesis [5]. However, if we allow for an independent variation in both m_e and α , then these limits become much less restrictive, since these two effects are nearly degenerate. For $\ell_{max} \sim 500 - 1000$ the limit on $|\Delta m_e/m_e|$ is no better than 10%, while for $\ell_{max} > 1500$ it can be as small as 10^{-2} . This is consistent with the results shown in Fig. 2: the degeneracy between the effect of changing m_e and the effect of changing α is broken only at the largest values of l . As we have noted, there are models in which simultaneous variation of $\langle \phi \rangle$ and α occurs “naturally” [4,7]. Hence, our result also supplies an important caveat to the limits on $|\Delta \alpha/\alpha|$ discussed in references [2,3]: these limits will apply only if the Higgs vacuum expectation value is taken to be constant.

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